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ABSTRACT

State and Federal Governments can provide aid to local school districts in many different forms, which, in general, will have different effects on school district spending. Policymakers, therefore, need to know what the impacts of alternative aid programs will be in order to best achieve their objectives. This report presents a method, based on a simple empirical model, for predicting the effect of outside aid on the educational expenditures of school districts. Unlike earlier studies, this method attempts to estimate the preference function that undergirds school district behavior, making it possible to predict impacts of all types of aid, inclusive of those not used in the past. (Author)

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A Method for Predicting the Effects of Different Forms of Outside Aid on Local Educational Expenditure

Edgar O. Olsen

A Report prepared for

THE FORD FOUNDATION

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PREFACE

This report is part of a series of Rand studies on federal, state, and local fiscal relations sponsored by The Ford Foundation. The first report in this series presents theoretical models to provide a foundation for empirical studies of the fiscal behavior of local public school districts and, ultimately, for efforts to predict impacts of alternative state and federal grant-in-aid programs (S. M. Barro, *Theoretical Models of School District Expenditure Determination and the Impacts of Grants-in-Aid*, The Rand Corporation, R-867-FF, February 1972). The present report is an empirical study, based on a simple theoretical model, which provides a method for predicting the effect of different forms of outside aid on the educational expenditure of a school district.

The author is currently an associate professor in the Department of Economics at the University of Virginia, Charlottesville, Virginia, and a consultant to The Rand Corporation.

SUMMARY

State and federal governments can provide aid to local school districts in many different forms, which, in general, will have different effects on school district spending. The purpose of this report is to present a method for predicting the effect of outside aid on the educational expenditure of any school district.

These predictions are based on a simple economic theory of school district decisionmaking. It is assumed that school district decisionmakers care about the quantity of educational services per student and the quantity of other goods per capita consumed by residents of the school district. It is also assumed that these decisionmakers have a budget constraint depending on (1) the disposable income of residents after paying taxes to all other political jurisdictions, (2) the prices of educational service and other goods, and (3) the amount and form of state and federal aid to the school district.

To move from the theory to a method for making predictions, we posit an indifference map with a specfic functional form and note that almost all state and federal aid to education during the time period under consideration was equivalent to unrestricted cash grants. These factors, together with the general assumptions of our theory, imply a specific relationship between educational expenditure, income, prices, number of students, population, and the amount of state and federal aid. From this relationship at the school district level, we deduce a relationship between variables aggregated to the state level. Estimates of the parameters of the indifference map are derived from estimates of the coefficients of the expenditure equation.

To predict the educational expenditure of a school district, we must note the school district's budget constraints, taking into account the form and amount of outside aid under consideration, and maximize the estimated utility function subject to these constraints. Since there are only two variables, the solution can be found by using a graph and making a few simple calculations. Illustrations are provided.

There are many empirical studies of the determinants of school

district expenditures, but none of these results are suitable for predicting the effects of a wide range of aid forms because they either ignore the effect of outside aid or implicitly assume that the *form* of aid makes no difference. This study differs in that it takes account of the effect of past aid on past expenditures and attempts to estimate the preference function that underlies school district behavior, thereby making it possible to estimate impacts of types of aid that have not been used in the past. Since we want to predict the effects of forms of aid with which there has been no experience, it is particularly important for the underlying theory of expenditure determination to be reliable. Therefore, we conducted a number of tests of our theory. The estimated coefficients in the expenditure relationship had the expected signs and were highly significant. The coefficient of determination was large. However, more powerful tests uncovered defects in the model. We conclude that the method used in this report would produce tolerably good predictions of the difference in educational expenditure attributable to different forms of outside aid, but that future empirical studies based on less metaphorical theories of school district decisionmaking would eventually permit better predictions.

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I. INTRODUCTION

School district revenues consist of grants-in-aid from state and local governments plus local taxes. The outside aid comes in many different forms--lump-sum grants, matching grants, grants that are conditional on local tax rates, and various combinations of the above. In general, both the amount of aid and the form in which aid is provided will affect expenditure and tax decisions by local school authorities. Policymakers at the state and federal levels, who are concerned about levels of school spending and distributions of expenditures and tax burdens among localities, provide aid precisely for the purpose of influencing those variables. But they need to be able to predict how the recipients will respond if they are to achieve their financial objectives. The primary purpose of this report is to help meet that need by providing a method for estimating the effects of different forms and amounts of aid on a school district's expenditures.

Because of its policy-oriented goal, this study differs from earlier studies of the determinants of school district expenditures in some important respects. The earlier results are not suitable for predicting the effects of a wide range of aid forms because they either ignore the effect of outside aid entirely or implicitly assume that only the amount, but not the form of aid, makes a difference. This study, in contrast, attempts to estimate the preference function that underlies school district behavior, thereby making it possible to predict impacts of all types of aid, including types that have not been used in the past. In order to do this, the analysis takes account of the effects on school expenditures of relevant price variables, which have generally been omitted in earlier work.

The organization of the report is as follows:

Section II presents the basic theory of school district spending, first in deterministic form, then in stochastic form, and goes on to show how a statistical relationship at the state level may be derived from the original relationship at the local district level.

Section III presents the principal empirical results, including the estimated expenditure and demand functions and the underlying preference function. The latter is inferred from the estimated expenditure equation.

Section IV, which is primarily directed to potential users of the predictive model, presents a set of examples showing how the results may be used to estimate what local districts will spend when they receive different forms of aid. The cases considered are a cash grant with a minimum educational expenditure constraint and a matching grant.

Section V, which is primarily directed to persons engaged in research, tests some of the statistical assumptions underlying the empirical analysis and some of the implications of the theory. These tests are more rigorous than those usually applied to models of this kind, but since the results could be used to predict effects of forms of aid with which there has been no experience, it is particularly important that the underlying theory be verified.

Finally, Section VI presents an overall evaluation of the model and some suggestions for further research.

II. DEVELOPMENT OF THE MODEL

This section has three parts. First, we outline the theory of school board decisionmaking underlying our empirical work. Second, we state our assumptions regarding the indifference map, the budget constraint of school districts, and the stochastic relationship at the school district level that will be used to estimate the parameters of this indifference map. Third, we deduce a statistical relationship at the state level from the statistical relationship at the school level.

THE BASIC THEORY

Decisions concerning spending on public education are made by elected officials and directly by the electorate. Voters are concerned about the quantities of educational service and other goods that they consume.

Therefore, it is assumed that the school board is concerned about the quantity of educational goods (service) per student Q_e/A and the quantity of noneducational goods per capita Q_x/N consumed by residents of the school district. It is also assumed that the school board has an indifference map, such as the one depicted in Fig. 1, expressing its preferences for various combinations of the two goods. That is, the school board is assumed to have an ordinal preference function of the form

$$U = W\left(\frac{Q_e}{A}, \frac{Q_x}{N}\right) .$$

The market price of providing one unit of education to each student is equal to the market price per unit of education multiplied by the number of students. Similarly, the price of providing one unit of other goods to each person is equal to the market price of noneducation goods multiplied by the number of people. The school board is assumed to face a budget constraint determined by the personal disposable income of residents of the local school district Y, the market prices P_eA and P_xN of the two goods, and the level and form of outside aid to the school district. In the absence of outside aid, the school

board's budget constraint is assumed to be

$$P_e A\left(\frac{Q_e}{A}\right) + P_x N\left(\frac{Q_x}{N}\right) = Y \quad ,$$

which may be rewritten as

$$P_e Q_e + P_x Q_x = Y \quad .$$

This indicates that the school board's expenditure on education $P_e Q_e$ plus the residents' expenditure on other goods $P_x Q_x$ is equal to the aggregate personal disposable income in the school district. This budget constraint is shown by the straight line in Fig. 1.

Finally, we assume that the school board chooses a combination of goods that maximizes the ordinal utility function W subject to the budget constraint. If a school board had the preferences and faced

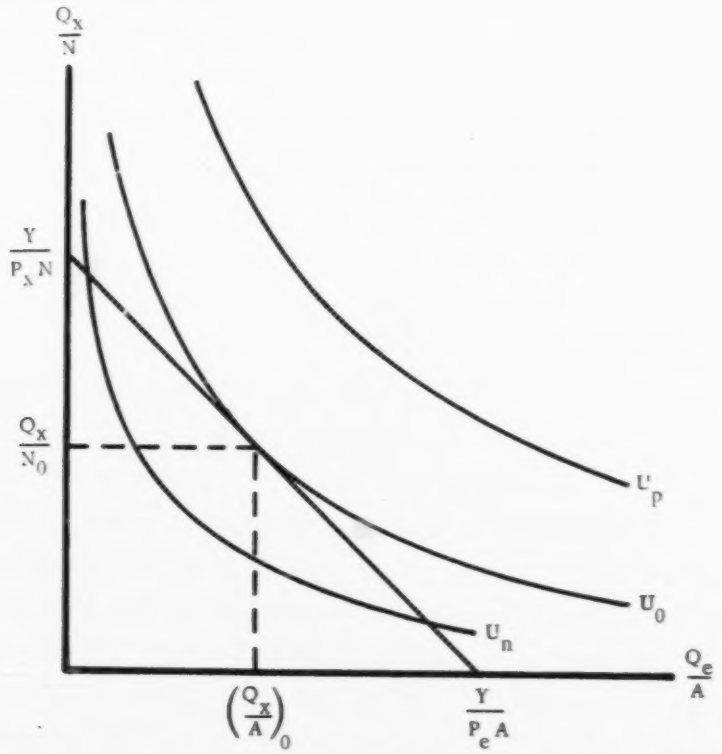

Fig. 1

the budget constraint depicted in Fig. 1, then it would choose to provide $(Q_e/A)_0$ units of educational service per student and spend $P_eA(Q_e/A)_0$ on education. This would leave the residents of the school district with $P_xN(Q_x/N)_0$ to spend on other goods.

FROM THEORY TO STOCHASTIC RELATIONSHIPS

Outside aid to a school district changes its budget constraint. Under the assumptions of the preceding subsection, we must have an estimate of a school district's indifference map in order to predict the effect of any change in its budget constraint on educational expenditure.

We assume that each school district has an indifference map of the form*

$$U = \left[\left(\frac{Q_e}{A}\right) - b_e\right]^{c_e} \left[\left(\frac{Q_x}{N}\right) - b_x\right]^{c_x} \qquad (1)$$

The problem is to estimate the parameters b_e, c_e, b_x, and c_x, which are assumed to be the same in all school districts. In other words, we assume that the residents of each school district will consume at least b_e units of educational service per student and b_x units of noneducation goods per capita, and that $100c_e$ percent of the school district's expenditure in excess of the amount necessary to buy the minimal combination (b_e, b_x) is devoted to education and $100c_x$ percent to other goods.

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*This indifference map underlies many of the complete systems of demand equations that have been estimated. For example, see R. Stone, "Linear Expenditure Systems and Demand Analysis: An Application to the Pattern of British Demand," *Economic Journal*, Vol. 64, September 1954, pp. 511-527; K. Yoshihara, "Demand Functions: An Application to the Japanese Expenditure Pattern," *Econometrica*, Vol. 37, April 1969, pp. 257-274; R. A. Pollak and T. J. Wales, "Estimation of the Linear Expenditure System," *Econometrica*, Vol. 37, October 1969, pp. 611-628; R. W. Parks, "Systems of Demand Equations: An Empirical Comparison of Alternative Functional Forms," *Econometrica*, Vol. 37, October 1969, pp. 629-650.

Now let us consider the budget constraints faced by school districts. The data for this study are obtained from the academic years 1953-54 through 1965-66. During this period, almost all state and federal aid to local school districts was equivalent in effect to an unrestricted cash grant to these districts. In essence, the school districts were given a certain amount of aid on the condition that they spend at least some other amount on education.* The minimum required expenditure on education determines a maximum amount of expenditure on noneducational goods by residents of school districts that accept outside aid. Hence, almost all school districts had budget spaces defined by the inequalities

$$\left[P_e A\left(\frac{Q_e}{A}\right) + P_x N\left(\frac{Q_x}{N}\right) \leq Y\right] \text{ or } \left\{\left[P_e A\left(\frac{Q_e}{A}\right) + P_x N\left(\frac{Q_x}{N}\right) \leq Y + S + F\right]\right.$$

$$\text{and} \quad \left.\left[P_x N\left(\frac{Q_x}{N}\right) \geq Y - L\right]\right\},$$

where S is the amount of state aid; F, the amount of federal aid; and L + S + F, the minimum educational expenditure required to obtain the outside aid. This budget space is represented by the shaded area in Fig. 2. Furthermore, almost all of these school districts spent more

*See C. S. Benson, *The Economics of Public Education*, 2d ed., Houghton Mifflin, 1968, pp. 146-151 and 154-222 (especially pp. 166, 214, 216). Wisconsin and Rhode Island had open-ended matching grants and New York, Massachusetts, and Maine had modified versions of such grants under which the state shared locally determined expenditures up to a stated maximum amount per pupil. In retrospect, it would have been better not to have used the data for these states during the years aid was given in this form. However, since some of these states used fixed unit equalizing grants during some of the period (e.g., Rhode Island did not adopt matching grants until 1960-61 and New York not until 1962-63) and many local school districts in New York, Massachusetts, and Maine spent more than enough to obtain the maximum amount of state aid (which implies that state aid to these districts was equivalent in effect to an unrestricted cash grant), the errors resulting from including this data are likely to be small.

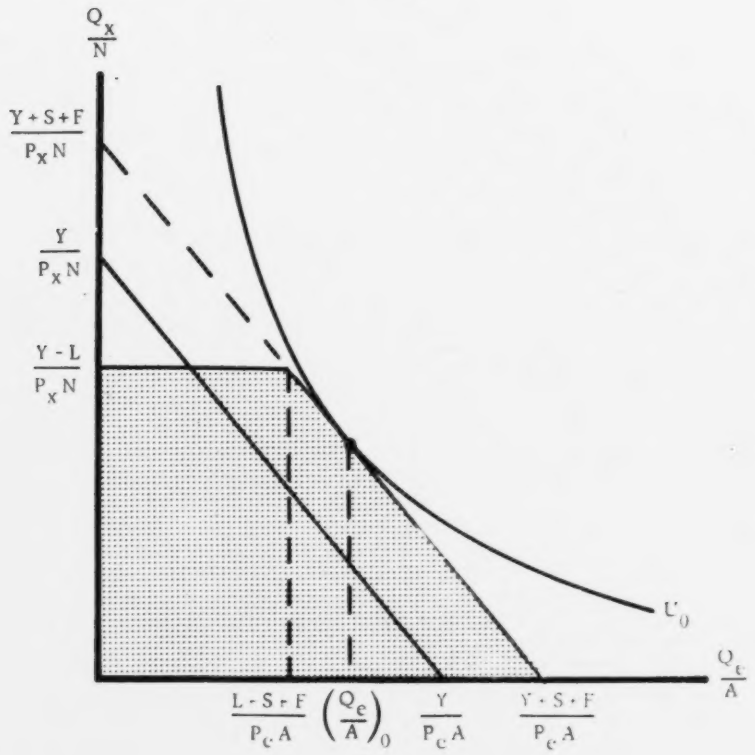

Fig. 2

on education than was required to obtain outside aid.* The relationship between the budget space and the preference map for such a school district is indicated in Fig. 2. The school district chooses to spend $P_e A(Q_e/A)_0$, which is greater than the minimum required expenditure $L + S + F$. Thus, this constraint is not binding, and we can proceed as if the school district's budget constraint were simply

$$P_e A\left(\frac{Q_e}{A}\right) + P_x N\left(\frac{Q_x}{N}\right) = Y + S + F \quad . \qquad (2)$$

This budget constraint effectively depicts the situation of the great majority of school districts during the period 1953-54 to 1965-66.

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*See Benson, op.cit., especially pp. 177 and 182; and New York State Education Department, Bureau of Educational Finance Research, *The Urban Education Crisis: Some Fiscal Considerations*, Studies of Public School Support, 1967 Series, Albany, New York, February 1968, p. 13.

Maximizing the ordinal preference function, Eq. (1), subject to the budget constraint, Eq. (2), yields the expenditure function

$$P_e Q_e = c_e (Y + S + F) + (1 - c_e) b_e P_e A - c_e b_x P_x N \quad . \tag{3}$$

Our working hypothesis is that the underlying stochastic model explaining variations in expenditure is

$$P_e Q_e = \alpha_0 (Y + S + F) + \alpha_1 P_e A + \alpha_2 P_x N + U \quad . \tag{4}$$

$$U \sim N(0, \sigma^2) \quad . \tag{5}$$

We also assume that, given a sample of T joint observations $(P_e Q_e)_t$, $(Y + S + F)_t$ $(P_e A)_t$, and $(P_x N)_t$ [t = 1, .., T] produced by the model (Eqs. (4) and (5)), the disturbances U_t [t = 1, ..., T] are mutually independent and $(Y + S + F)_t$, $(P_e A)_t$, and $(P_x N)_t$ are nonstochastic.

However, an examination of the residuals from this regression (reported in Sec. V) suggests that the variance of the error term in Eq. (4) is not constant as assumed and, in fact, varies directly with income. This finding is consistent with the findings of many other studies of consumer behavior. As a result, we replace Eq. (4) by Eq. (6), as follows:

$$P_e Q_e = \alpha_0 (Y + S + F) + \alpha_1 P_e A + \alpha_2 P_x N + U\sqrt{Y + S + F} \quad . \tag{6}$$

In Sec. III, estimates of the parameters of the indifference map, Eq. (1), are obtained from estimates of the parameters of the expenditure function, Eq. (6).

FROM A STATISTICAL RELATIONSHIP AT THE SCHOOL DISTRICT LEVEL TO A STATISTICAL RELATIONSHIP AT THE STATE LEVEL

Our data base consists of aggregate data by state. However, since our theory concerns decisionmaking at the school district level, we would like to estimate a relationship consistent with some plausible behavior function at this level. More importantly, since we want to be able

to predict the effect of different forms of aid on educational expenditure by school district, we must estimate a relationship with the aggregate data that permits us to derive an indifference map of school districts. We now deduce a statistical relationship at the state level from the statistical relationship at the school district level of the preceding subsection. Let us rewrite Eq. (6) with subscripts as follows:

$$E_{dst} = \alpha_0 (Y + S + F)_{dst} + \alpha_1 P_{e,t} A_{dst} + \alpha_2 P_{x,t} N_{dst} + U_{dst} \sqrt{(Y + S + F)_{dst}} \quad . \tag{7}$$

The subscripts indicate the d^{th} district in the s^{th} state in the t^{th} time period, We assume that the prices of educational and noneducational goods vary with time, but not geographically. To develop respectable price indices that vary geographically is far beyond the scope of this report.

We do not have data to estimate Eq. (7) as it stands. However, we can deduce a stochastic relationship from this equation with the same parameters as Eq. (7).

Summing over all school districts in a state, we obtain

$$\sum_{d=1}^{D_{st}} d_{dst} = \alpha_0 \sum_{d=1}^{D_{st}} (Y + S + F)_{dst} + \alpha_1 P_{e,t} \sum_{d=1}^{D_{st}} A_{dst} + \alpha_2 P_{x,t} \sum_{d=1}^{D_{st}} N_{dst} + \sum_{d=1}^{D_{st}} U_{dst} \sqrt{(Y + S + F)_{dst}} \quad ,$$

which can be written more compactly as

$$E_{st} = \alpha_0 (Y + S + F)_{st} + \alpha_1 P_{e,t} A_{st} + \alpha_2 P_{x,t} N_{st} + \sum_{d=1}^{D_{st}} U_{dst} \sqrt{(Y + S + F)_{dst}} \quad . \tag{8}$$

Dividing both sides of the equation by $\sqrt{(Y + S + F)_{st}}$, we derive

$$\frac{E_{st}}{\sqrt{(Y + S + F)_{st}}} = \alpha_0 \sqrt{(Y + S + F)_{st}} + \alpha_1 \frac{P_{e,t} A_{st}}{\sqrt{(Y + S + F)_{st}}} + \alpha_2 \frac{P_{x,t} N_{st}}{\sqrt{(Y + S + F)_{st}}}$$

$$+ \frac{\Sigma U_{dst} \sqrt{(Y + S + F)_{dst}}}{\sqrt{(Y + S + F)_{st}}} . \qquad (9)$$

If the variance of U_{dst} is constant, then the variance of the error term in Eq. (9) is also constant because

$$\text{var} \frac{\sum_{d=1}^{D_{st}} U_{dst} \sqrt{(Y + S + F)_{dst}}}{\sqrt{(Y + S + F)_{st}}} = \frac{\sum_{d=1}^{D_{st}} \text{var} \left(U_{dst} \sqrt{(Y + S + F)_{dst}} \right)}{(Y + S + F)_{st}}$$

$$= \frac{\sum_{d=1}^{D_{st}} (Y + S + F)_{dst} \text{ var } U_{dst}}{(Y + S + F)_{st}}$$

$$= \frac{\sigma^2 \sum_{d=1}^{D_{st}} (Y + S + F)_{dst}}{(Y + S + F)_{st}}$$

$$= \frac{\sigma^2 (Y + S + F)_{st}}{(Y + S + F)_{st}}$$

$$= \sigma^2$$

Our predictions are based on estimates of the parameters of Eq. (9).

III. ESTIMATION OF THE MODEL

DATA SOURCES

This report is based on school district data aggregated to the state level for all states except Alaska and Hawaii. From the U.S. Department of Health, Education, and Welfare, we obtained data on educational expenditure, state and federal aid to local school districts, population, and the number of public school students and school districts.* We used Lorne Wollatt's national cost-of-education index and the Bureau of Labor Statistics' national Consumer Price Index (CPI).** The Office of Business Economics of the Department of Commerce provided us with personal disposable income by state.† A few words about some of these data are in order.

The expenditure data do not accurately reflect the value of the resources expended on education because they do not include the rental value of the property and capital equipment used by the public schools. The *Biennial Survey of Education* and the *Statistics of State School Systems* contain rough estimates of the stock value of school property owned by local basic administrative units. In most cases, the estimate is the unweighted sum of original costs and the costs of all additions and alterations. In some cases, it is the value for which the property is insured or an estimate of the replacement cost. An estimate of the

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*U.S. Department of Health, Education, and Welfare, Office of Education, National Center for Educational Statistics, *Biennial Survey of Education: State School Systems* for 1953-54 through 1957-58, and *Statistics of State School Systems* for later years.

**L. H. Wollatt, "The Cost of Education Index 1939-1958" (mimeographed), Bureau of Research, Baltimore Public Schools, Baltimore, Maryland, December 1958; and U.S. Department of Labor Statistics, *Monthly Labor Review*, various issues.

†R. B. Bretzfelder, Q. F. Dallavalle, and D. A. Hirschberg, "Personal Income, 1968, and Disposable Income, 1929-1968, by States and Regions," *Survey of Current Business*, April 1969, pp. 16-32.

flow value of school property in each state could be obtained from these stock values, and this estimate added to current expenditure to obtain an estimate of the value of all resources used in the production of educational services. Early in the study when the issues were less clear, we decided to follow the lead of others in this field and use data on current expenditure. Hence, our predictions reflect current expenditures of school districts. In retrospect, it is clear that we should have at least experimented with a total resource cost variable.

The data on personal disposable income correspond to our theoretical concept quite well. Income taxes are subtracted and cash grants from governments to individuals are added.

Data on the number of school districts are not available for some states in some years. We obtained 15 of the 336 numbers by interpolation.

Wollatt's cost-of-education index refers only to current expenditures. Furthermore, the index does not vary among states. Hence, if we use this index we assume that the cost of producing a unit of educational service is the same in all school districts. Furthermore, Wollatt's index suffers from the weakness of most price indices. It does not take full account of quality changes. There has been some work done on adjusting price indices for quality differences.* However, it was beyond the resources of this project to produce better price-of-education indices. Wollatt's index extended through the 1962-63 academic year; we extended his index through the 1968-69 academic year and corrected the value of his index in 1962-63. Table 1 displays the values of Wollatt's index for the years of our sample and the values of the equivalent series of index numbers actually used in the calculations. It is this latter series of index numbers that *must* be used in making our predictions.

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*For example, see Z. Griliches, "Hedonic Price Indexes for Automobiles: An Econometric Analysis of Quality Change," in *The Price Statistics of the Federal Government*, National Bureau of Economic Research, New York, 1961.

Table 1

WOLLATT'S COST-OF-EDUCATION INDEX

School Year	Original Index	Transformed Index
1953-1954	128.2	1.000
1955-1956	142.3	1.110
1957-1958	156.2	1.218
1959-1960	166.8	1.301
1961-1962	181.0	1.412
1963-1964	193.6	1.510
1965-1966	204.4	1.594

The CPI is used as an index of the price of noneducational goods. The prices used to construct the CPI include indirect taxes. Therefore, these taxes are correctly accounted for. However, the CPI does not take account of transfers from governments to individuals that effectively reduce the price of noneducational goods. For our purposes this is a defect, but we do not regard it as an important factor. In principle, we should have removed the influence of the price of education from the CPI, but we consider this influence to be so small that we did not make any correction on this account. Table 2 displays the values of the CPI for the years of our sample and for the values of the equivalent series of index numbers actually used in the calculations. This latter index *must* be used in making predictions based on the empirical results of this report.

Table 2

CONSUMER PRICE INDEX

School Year	Original Index	Transformed Index
1953-1954	93.2	1.000
1955-1956	93.3	1.001
1957-1958	98.0	1.052
1959-1960	101.5	1.089
1961-1962	104.2	1.118
1963-1964	106.7	1.145
1965-1966	109.9	1.179

EXPENDITURE AND DEMAND FUNCTIONS

We now estimate the parameters of the statistical model described in Eqs. (7) through (9) and infer estimates of the parameters of the indifference map from them.

We estimate Eq. (9) by the method of least squares to obtain

$$\frac{E}{\sqrt{Y + S + F}} = \underset{(41.7)}{.044\sqrt{Y + S + F}} + \underset{(28.0)}{277\frac{P_e A}{\sqrt{Y + S + F}}} - \underset{(-23.9)}{63.2\frac{P_x N}{\sqrt{Y + S + F}}}$$

$$R^2 = .96 \qquad s^2 = 107{,}700 \; . \tag{10}$$

The numbers in parentheses beneath the estimated coefficients are t-scores. All variables except P_e and P_x are measured in natural units. These price indices were given the value 1 in academic year 1953-54.

The implied expenditure function at the school district level is

$$E = .044(Y + S + F) + 227P_e A - 63.2P_x N \; , \tag{11}$$

and the implied demand function is

$$\frac{Q_e}{A} = 227 + .044\left[\frac{(Y + S + F)}{P_e A}\right] - 63.2\left(\frac{P_x N}{P_e A}\right) \; , \tag{12}$$

where S and F are state and federal aid equivalent to unrestricted cash grants.

From our discussion on page 5, it is clear that we expect the parameters of the indifference map to satisfy the following inequalities:

$$b_e > 0 \quad b_x > 0 \quad 0 < c_e < 1 \quad 0 < c_x < 1 \; .$$

Looking at the relationship between the parameters of the indifference map and the parameters of the stochastic relationship, Eq. (9), we see that the preceding inequalities imply that

$$0 < \alpha_0 < 1 \quad \alpha_1 > 0 \quad \alpha_2 < 0 \; .$$

We limit our analysis to the following four hypotheses:

$$H_0: \alpha_0 = 0 \qquad H_0: \alpha_0 = 1$$
$$H_1: \alpha_0 > 0 \qquad H_1: \alpha_0 < 1$$

$$H_0: \alpha_1 = 0 \qquad H_0: \alpha_2 = 0$$
$$H_1: \alpha_1 > 0 \qquad H_1: \alpha_2 < 0$$

We expect to reject the null hypothesis in each case, and our tests do so easily at the .005 level of significance. These tests provide impressive support for the model underlying our estimated indifference map.

These results confirm what many other studies of educational spending have shown. Education is a normal good. The greater the income of a community, the greater the quantity of educational services demanded. Furthermore, it confirms the widely held belief that the demand for educational services is price-inelastic. According to Eq. (11), the higher the price of educational service to a school district, the greater the expenditure on education will be.

These equations can be used directly to predict money and real expenditure in the absence of outside aid or in the presence of outside aid which is equivalent to an unrestricted cash grant or to an open-ended matching grant. In order to predict these effects in the presence of many other forms of aid, we must know the recipient's indifference map.

THE INFERRED INDIFFERENCE MAP

The parameters of the indifference map, Eq. (1), can be deduced from the parameters of the stochastic model, Eq. (5) and (6). Comparing Eqs. (3) and (6) we see that

$$\alpha_0 = c_e ,$$
$$\alpha_1 = (1 - c_e)b_e ,$$
$$\alpha_2 = -c_e b_e .$$

In order that the amounts spent on the two goods add up to total expenditure,

$$c_x = 1 - c_e .$$

Thus,

$$c_e = \alpha_0 ,$$

$$b_e = \alpha_1/(1 - \alpha_0) ,$$

$$c_x = 1 - \alpha_0 ,$$

$$b_x = -\alpha_2/\alpha_0 .$$

Therefore, if we knew the population values of the parameters of the stochastic model, we could deduce the parameters of the indifference map.

Our estimators of the parameters of the indifference map are

$$\hat{c}_e = \hat{\alpha}_0 ,$$

$$\hat{b}_e = \hat{\alpha}_1/(1 - \hat{\alpha}_0) ,$$

$$\hat{c}_x = 1 - \hat{\alpha}_0 ,$$

$$\hat{b}_x = -\hat{\alpha}_2/\hat{\alpha}_0 ,$$

where the $\hat{\alpha}$'s are least-squares estimators of the parameters of the stochastic model. There are estimators of the parameters of the indifference map with more desirable properties than these estimators.*

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*See R. W. Parks, "Systems of Demand Equations: An Empirical Comparison of Alternative Functional Forms," *Econometrica*, Vol. 37, October 1969, pp. 632-633 and 642-644.

However, the additional work necessary to obtain these estimates did not seem worthwhile.

From the least-squares estimates of the α's reported in Eq. (10), we conclude that the indifference map of school districts is

$$U = \left[\left(\frac{Q_e}{A}\right) - 237\right]^{.044} \left[\left(\frac{Q_x}{N}\right) - 1437\right]^{.956} . \qquad (13)$$

IV. PREDICTION OF THE EFFECT ON EDUCATIONAL EXPENDITURE OF CHANGES IN THE FORM OF OUTSIDE AID

To predict the educational expenditure of a school district, we maximize the ordinal preference function, Eq. (13), subject to the school district's budget constraint, which will depend in part on the forms and amounts of outside aid available. This is a nonlinear programming problem. However, since there are only two variables, the solution can be found using a graph and making a few calculations in all cases of practical interest. The purpose of this section is to illustrate this simple method for predicting the educational expenditure of a school district.

Initially, let us assume that there is no outside aid to education. Governments at all levels have decided how much of each good and service other than education to provide and have collected taxes to pay for these goods and services. Each individual in the school district has some income after these taxes. The budget constraint facing the school district is

$$P_e A\left(\frac{Q_e}{A}\right) + P_x N\left(\frac{Q_x}{N}\right) = Y \ . \tag{14}$$

In a particular case, we could predict the educational expenditure for a school district by substituting its average school attendance, population, personal disposable income, and the values of the price indices for the particular year into Eq. (14) and maximize Eq. (13) subject to this constraint. However, the answer can be obtained more easily by substituting these numbers into the following:

$$E = .044Y + 227P_e A - 63.2P_x N \ .$$

Therefore, we can predict educational expenditure per pupil using the formula:

$$\frac{E}{A} = .044\left(\frac{Y}{N}\right)\left(\frac{N}{A}\right) + 227P_e - 63.2P_x\left(\frac{N}{A}\right) \ . \tag{15}$$

For example, our prediction of the educational expenditure per pupil in a school district with one-fifth of its population in school and a personal disposable income of $2000 per capita in 1965 is

$$.044(2000)(5) + 227(1.594) - 63.2(1.179)(5) = \$429 .^{*}$$

CASH GRANT WITH MINIMUM EDUCATIONAL EXPENDITURE CONSTRAINT

Now let us suppose that both the federal and state governments decide to initiate programs of aid to local school boards. To finance these programs, federal and state income tax rates are raised. Therefore, one effect of these programs is to reduce the personal disposable income of residents of the school district by some amount T. In return, the school district receives an amount S from the state government and F from the federal government. Let us assume that the only restriction on the use of this money is that it all be spend on education. Therefore, the residents of the school district can spend no more than Y - T on other goods. If the school district were to refuse this outside aid, its budget constraint would be

$$P_e A\left(\frac{Q_e}{A}\right) + P_x N\left(\frac{Q_x}{N}\right) \leq Y - T . \tag{16}$$

If the school district were to accept outside aid, its budget constraint would be

$$P_e A\left(\frac{Q_e}{A}\right) + P_x N\left(\frac{Q_x}{N}\right) \leq Y + S + F - T .$$

and

(17)

$$P_x N\left(\frac{Q_x}{N}\right) \leq Y - T .$$

---

*In making these predictions it is essential that we use the price-of-education index and CPI on which our empirical results are based. In 1965, the values of our price-of-education index and CPI were 1.594 and 1.179, respectively.

Unless S + F - T is zero for all school districts, it must be positive for some districts, negative for others, and perhaps zero for still others. The striped areas in Figs. 3, 4, and 5 depict the budget constraints faced by the school district in these three cases. The dotted areas in these figures show the budget spaces of the school district in the absence of the federal and state programs.

Figure 3 clearly indicates that if the school district receives more state and federal aid to education than it pays in taxes to support the programs, then these programs add some combinations of goods to the original budget space, but subtract other combinations. In this case, it is possible that the school district prefers one of the combinations gained to any of the combinations lost. However, it is also possible that the school district prefers one of the combinations lost to any of the combinations gained, in which case the residents of the school district would favor the termination of these federal and state programs. However, since the school district must pay taxes regardless of whether or not it accepts outside aid, and since accepting outside aid permits it to have *at least one* combination of goods preferred to any combination attainable if aid is refused, the school district will accept the outside aid. This result also applies to the situations depicted in Figs. 4 and 5. That is, even a school district that opposes continuation of the programs will accept the aid.

Figure 4 shows that a school district that receives less federal and state aid than it pays in taxes to support these programs loses some combinations of goods, including the combination that would be chosen in the absence of the federal and state programs, and gains nothing. All of these school districts are made worse off by the federal and state programs.

Figure 5 indicates that, if the school district receives exactly as much state and federal aid to education as it pays in taxes to support these programs, then these programs subtract some combinations of goods from the original budget space and add none. If the combination of goods that would be chosen in the absence of these federal and state programs is among the combinations lost, then the school district is made worse off by the existence of these programs. Otherwise,

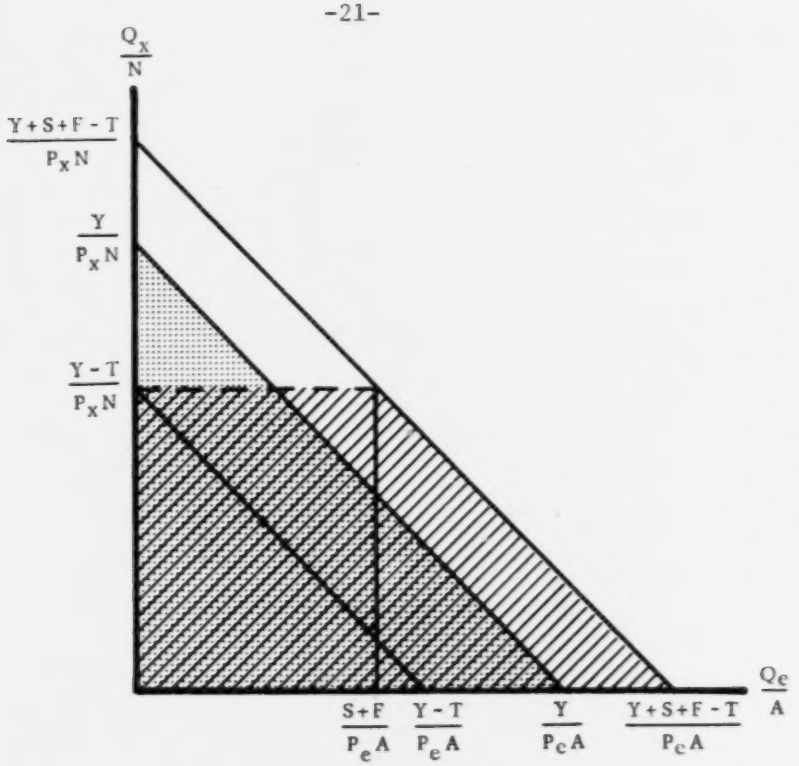
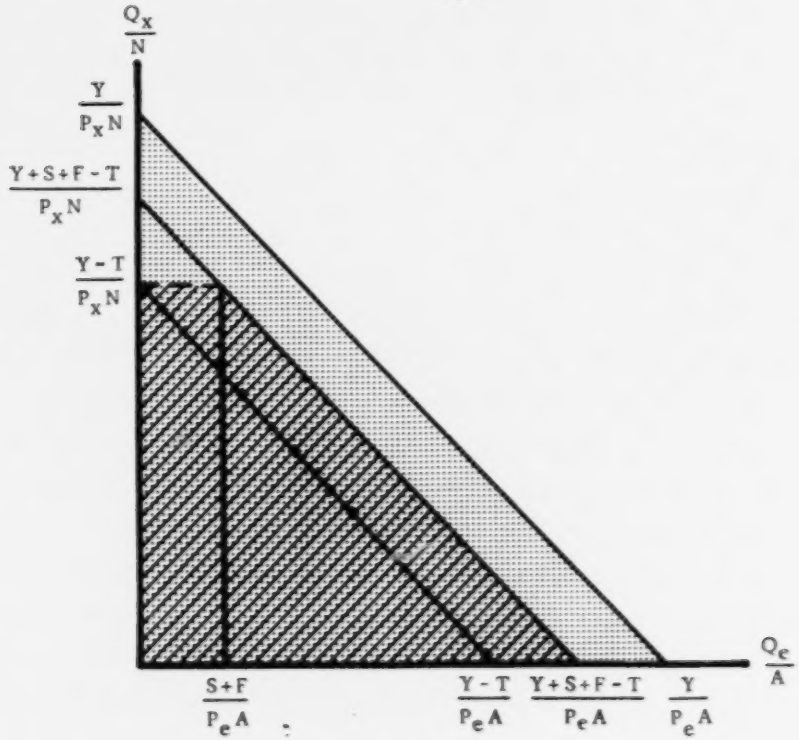

$\frac{Q_x}{N}$
$\frac{Y+S+F-T}{P_xN}$
$\frac{Y}{P_xN}$
$\frac{Y-T}{P_xN}$
$\frac{Q_e}{A}$
$\frac{S+F}{P_eA}$
$\frac{Y-T}{P_eA}$
$\frac{Y}{P_eA}$
$\frac{Y+S+F-T}{P_eA}$

Fig. 3


$\frac{Q_x}{N}$
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$\frac{Y-T}{P_xN}$
$\frac{Q_e}{A}$
$\frac{S+F}{P_eA}$
$\frac{Y-T}{P_eA}$
$\frac{Y+S+F-T}{P_eA}$
$\frac{Y}{P_eA}$

Fig. 4

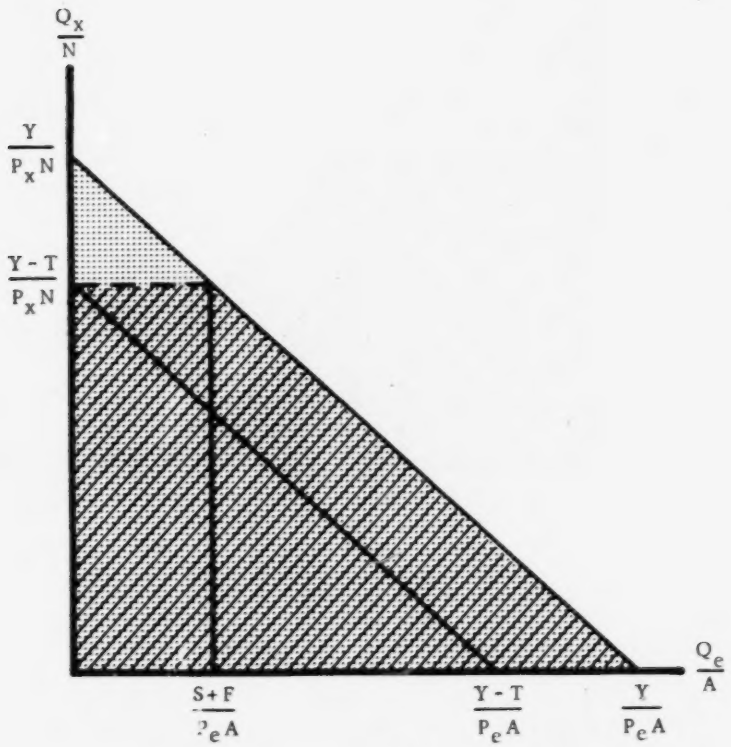

Fig. 5

the school district is not affected by them.

We have already shown how to predict the educational expenditure of a school district in the absence of programs of federal and state aid. To predict the effect of these programs on educational expenditure, we must also predict how much the school district will spend on education in the presence of these programs.

The dotted areas in Figs. 6 and 7 depict the budget constraint of a school district in the presence of the programs of state and federal aid considered in this subsection. This budget constraint has the same general appearance in all the cases depicted in Figs. 3, 4, and 5. The school board will select some combination of goods on the boundary of this budget space. The relationship between the budget space and the indifference map depicted in Fig. 6 indicates that

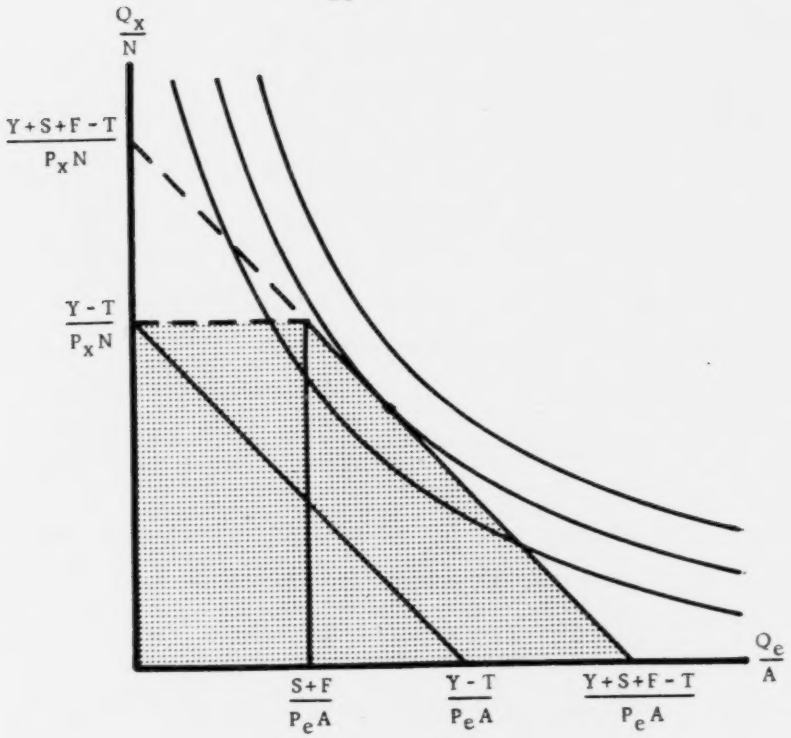
$\frac{Q_x}{N}$
$\frac{Y+S+F-T}{P_xN}$
$\frac{Y-T}{P_xN}$
$\frac{Q_e}{A}$
$\frac{S+F}{P_eA}$
$\frac{Y-T}{P_eA}$
$\frac{Y+S+F-T}{P_eA}$

Fig. 6

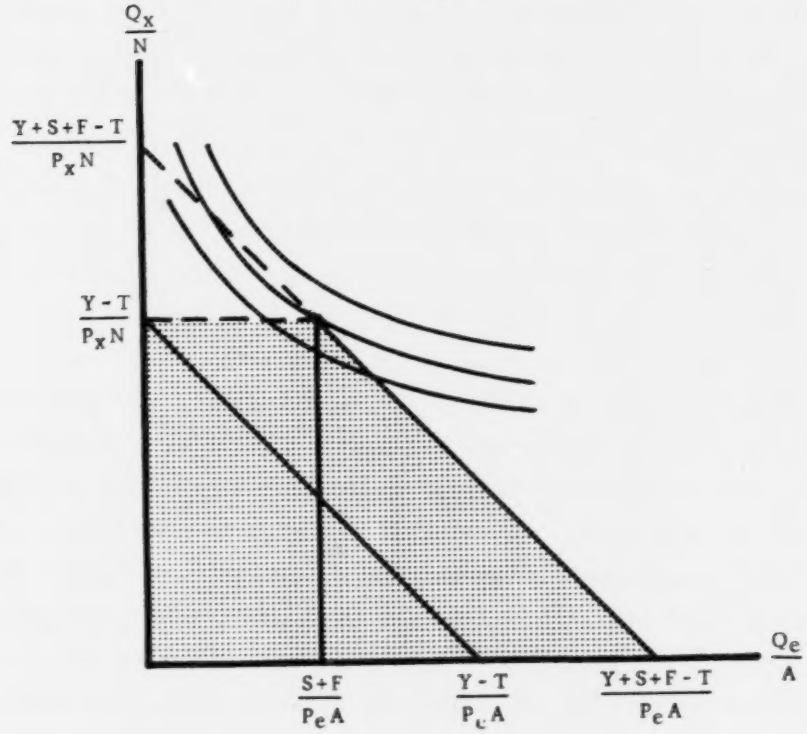
$\frac{Q_x}{N}$
$\frac{Y+S+F-T}{P_xN}$
$\frac{Y-T}{P_xN}$
$\frac{Q_e}{A}$
$\frac{S+F}{P_eA}$
$\frac{Y-T}{P_eA}$
$\frac{Y+S+F-T}{P_eA}$

Fig. 7

the school district would spend more than S + F on education even in the absence of the constraint. The constraint is not binding. The relationship between the budget space and the indifference map depicted in Fig. 7 shows that the school district would spend exactly the amount required on education as a condition for receiving the aid. To make our prediction, we first calculate the value of Eq. (18), as follows:

$$\frac{E}{A} = .044\left(\frac{Y + S + F - T}{N}\right)\left(\frac{N}{A}\right) + 227P_e - 63.2P_x\left(\frac{N}{A}\right). \qquad (18)$$

If this value is greater than (S + F)/A, then it is our prediction of educational expenditure per pupil. Otherwise, we predict that the school district will spend (S + F)/A per pupil on education. The difference between this prediction and our prediction of educational expenditure in the absence of these programs is our estimate of the effect of the programs on educational expenditure in the school district.

Suppose that the school district of the preceding example pays $40 per capita ($200 per pupil) in taxes to support these programs and receives $60 per capita ($300 per pupil) in state aid and $20 per capita ($100 per pupil) in federal aid to education. In this case, the value of Eq. (18) is $438, which exceeds $400, the amount of state and federal aid per pupil. Therefore, we predict that the school district would spend $438 per pupil on education in the presence of these programs of state and federal aid. That is, we predict that these programs result in an increase of $9 per pupil (= $438 - $429) in the amount spent by this school district on education. To illustrate the other possibility, suppose that state aid is $60 per capita ($300 per pupil); federal aid, $40 per capita ($200 per pupil); and tax, $60 per capita ($300 per pupil). In this case, the value of Eq. (18) is again $438, which is less than $500, the amount of federal and state aid per pupil. Therefore, we predict that the school district will spend $500 per pupil on education, which means that the federal and state programs have resulted in an increase of $71 per pupil (= $500 - $429) in educational expenditure.

The conclusions of this subsection can be easily generalized to cover cases in which the minimum educational expenditure of school districts accepting aid is different from the amount of outside aid.

MATCHING GRANT

Let us now vary our assumption about the form of outside aid. Assume that the federal government decides to pay the local school district a certain percentage R_f of its expenditure on education and the state government decides to pay a certain percentage R_s. If the school district rejects this aid, then its budget space is

$$P_e A\left(\frac{Q_e}{A}\right) + P_x N\left(\frac{Q_x}{N}\right) \leq Y - T . \tag{19}$$

If the school district accepts the aid, then its budget constraint is

$$P_e A\left(\frac{Q_e}{A}\right) + P_x N\left(\frac{Q_x}{N}\right) \leq Y - T + (R_f + R_s)P_e A\left(\frac{Q_e}{A}\right), \tag{20}$$

which can be rewritten as

$$(1 - R_f - R_s)P_e A\left(\frac{Q_e}{A}\right) + P_x N\left(\frac{Q_x}{N}\right) \leq Y - T . \tag{21}$$

Comparing Eqs. (14) and (21), we see that these federal and state programs reduce the price per unit of educational service to families in the school district from P_e to $(1 - R_f - R_s)P_e$ and reduce their disposable income from Y to Y - T.

The dotted areas in Figs. 8 and 9 depict the set of feasible combinations of goods in the absence of programs of outside aid; the striped areas depict the set of feasible combinations in the presence of programs of matching grants. Figure 8 depicts a case in which the matching grant subtracts some combinations of goods and adds others to the school district's budget space. If one of the additional combinations is preferred to all the lost combinations, then the residents of the school district gain from the program; otherwise, they lose. Figure 9 depicts a case in which the matching grant subtracts some combinations of goods and adds none. In this case, residents of the school district prefer no program to this one. However, comparing

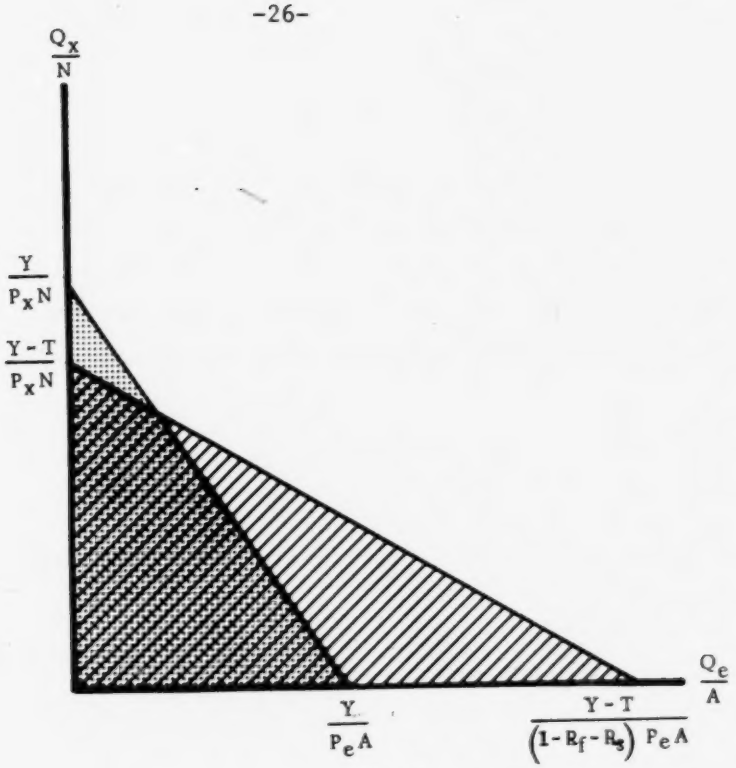
$\frac{Q_x}{N}$
$\frac{Y}{P_x N}$
$\frac{Y-T}{P_x N}$
$\frac{Q_e}{A}$
$\frac{Y}{P_e A}$
$\frac{Y-T}{(1-R_f-R_s)\,P_e A}$

Fig. 8

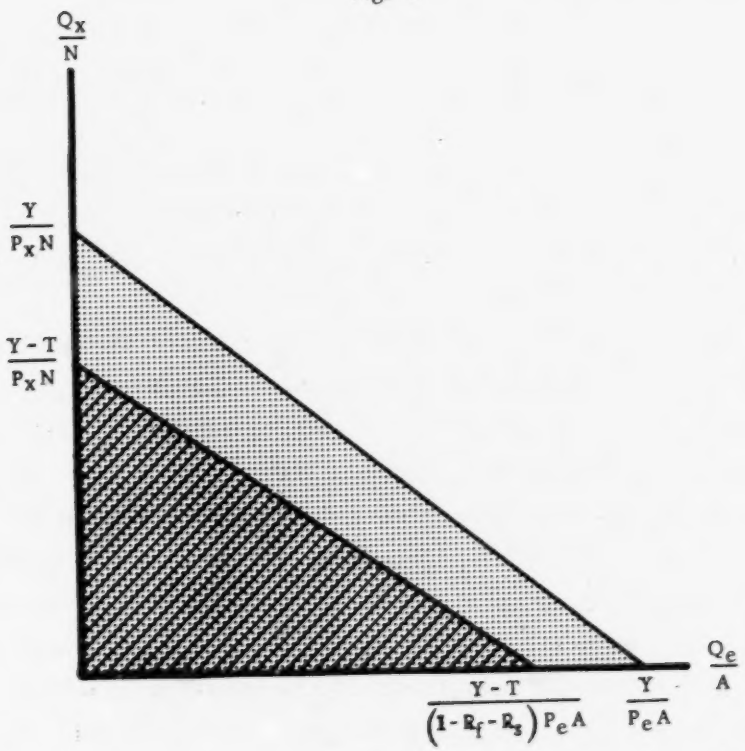
$\frac{Q_x}{N}$
$\frac{Y}{P_x N}$
$\frac{Y-T}{P_x N}$
$\frac{Q_e}{A}$
$\frac{Y-T}{(1-R_f-R_s)\,P_e A}$
$\frac{Y}{P_e A}$

Fig. 9

Eqs. (19) and (20), we see that no school district will reject aid of this kind.

In the presence of these programs of matching grants, we predict that a school board will collect

$$.044\left(\frac{Y-T}{N}\right)\left(\frac{N}{A}\right) + 227(1 - R_p - R_s)P_e - 63.2P_x\left(\frac{N}{A}\right) \qquad (22)$$

per pupil locally to support education. The federal and state governments will contribute $R_f E/A$ and $R_s E/A$, respectively. Therefore, expenditure per pupil will be

$$\frac{E}{A} = \frac{.044((Y-T)/N)(N/A) + 227(1 - R_f - R_s)P_e - 63.2P_x(N/A)}{1 - R_f - R_s} . \qquad (23)$$

Suppose that we are considering matching grants as alternatives to the lump-sum grants of the preceding two examples. To facilitate the comparisons, we select matching ratios that lead to the same costs to the state and federal governments.

To compare the lump-sum grants where the constraint is not binding with matching grants, assume that the federal government agrees to pay 17.8 percent and the state government 53.2 percent of the total educational expenditure of the school district. The school district must pay the remaining 29 percent. In this case, we predict that the per pupil educational expenditure of the school district would be

$$\frac{.044(1960)(5) + 227(.29)(1.594) - 63.2(1.179)(5)}{.29} = \$564 .$$

This school district receives \$100 per pupil in federal aid and \$300 per pupil in state aid. It levies \$164 per pupil in local taxes. Recall that in the absence of outside aid we predict that this school district would spend \$429 per pupil on education. Therefore, we predict that this program of matching grants would induce this school district to spend \$135 per pupil more on education. (Of course, since the estimated demand curve is

everywhere inelastic, the amount of money raised by local taxes to support education would fall, in this case, from $429 per pupil to $164 per pupil.) The lump-sum grants with a nonbinding constraint result in only $9 per pupil in additional educational expenditure. That is, we predict that the matching grants will result in a 31 percent [= 100(135/429)] increase in real educational expenditure, while the lump-sum grants with the same cost to state and federal governments will result in an increase of only 2 percent [= 100(8/429)]. In this particular case, it seems likely that the matching grants will be much more stimulative than lump-sum grants.

To compare the lump-sum grants where the constraint is binding with matching grants, assume that the federal government agrees to pay 31.9 percent, and the state government 47.8 percent of the total educational expenditure of the school district. The school district must pay the remaining 20.3 percent. In this case, we predict that the total educational expenditure of the school district will be

$$\frac{.044(1940)(5) + 227(.203)(1.594) - 63.2(1.179)(5)}{.203} = \$629 .$$

This school district receives $200 per pupil in federal aid and $300 per pupil in state aid. It levies $129 per pupil in local taxes. (Again, since the demand curve is everywhere inelastic, the amount of money raised by local taxes to support education would fall, in this case, from $429 per pupil to $129 per pupil.) Therefore, we predict that this school district would spend $200 per pupil more on education than it would in the absence of federal and state programs of matching aid. The lump-sum grants with a binding constraint would result in $71 per pupil in additional educational expenditure. That is, we predict that the matching grants will result in a 47 percent [= 100(200/429)] increase in real educational expenditure, while the lump-sum grants with the same cost to state and federal governments will result in an increase of only 17 percent [= 100(71/429)]. Again, it appears that the form of outside aid makes quite a difference.

It is possible to use the empirical results of this report to predict the effect of many other forms of outside aid on educational

expenditure in a school district. It is hoped that the illustrations of this section will be helpful to users analyzing the effects of more complicated aid schemes. Section V will be of interest primarily to persons engaged in research.

V. TESTS OF ASSUMPTIONS AND IMPLICATIONS

TESTS OF SOME ASSUMPTIONS OF THE STOCHASTIC MODELS

Since the desirable properties of least-squares estimators depend on the validity of certain assumptions, we now subject some of these assumptions to tests. These tests are conducted for our original model, Eqs. (4) and (5), and for our revised model, Eqs. (5) and (6).

The Original Model

In this subsection, the parameters of the statistical model, Eqs. (4) and (5), are estimated; estimates of the parameters of the indifference map, Eq. (1), are inferred from them; and tests of some assumptions underlying the desirable properties of least-squares estimators are conducted.

Let us rewrite Eq. (4) with the subscripts as follows:

$$E_{dst} = \alpha_0 (Y + S + F)_{dst} + \alpha_1 P_{e,t} A_{dst} + \alpha_2 P_{x,t} N_{dst} + U_{dst} . \quad (24)$$

We do not have data to estimate Eq. (24) as it stands. However, we may deduce from this equation a stochastic relationship with the same parameters as Eq. (24). First, we sum both sides of Eq. (24) over all school districts in a state to obtain

$$\sum_{d=1}^{D_{st}} E_{dst} = \alpha_0 \sum_{d=1}^{D_{st}} (Y + S + F)_{dst} + \alpha_1 P_{e,t} \sum_{d=1}^{D_{st}} A_{dst}$$

$$+ \alpha_2 P_{x,t} \sum_{d=1}^{D_{st}} N_{dst} + \sum_{d=1}^{D_{st}} U_{dst} ,$$

which can be written more compactly as

$$E_{st} = \alpha_0 (Y + S + F)_{st} + \alpha_1 P_{e,t} A_{st} + \alpha_2 P_{x,t} N_{st} + U_{st} . \quad (25)$$

This expression contains only variables that are available. However, if the error term U_{dst} satisfies the assumptions stated in Eq. (5), then the error term U_{st} does not, because

$$\text{var } U_{st} = \text{var} \sum_{d=1}^{D_{st}} U_{dst} = \sum_{d=1}^{D_{st}} \text{var } U_{dst} = \sum_{d=1}^{D_{st}} \sigma^2 = D_{st}\sigma^2 .$$

Fortunately, Eq. (25) can be manipulated into an equivalent expression in which the error term does satisfy the Gauss-Markov conditions by dividing both sides by $\sqrt{D_{st}}$, as follows:

$$\frac{E}{\sqrt{D_{st}}} = \alpha_0 \left[\frac{(Y + S + F)_{st}}{\sqrt{D_{st}}} \right] + \alpha_1 \left(\frac{P_{e,t} A_{st}}{\sqrt{D_{st}}} \right) + \alpha_2 \left(\frac{P_{x,t} N_{st}}{\sqrt{D_{st}}} \right) + \frac{U_{st}}{\sqrt{D_{st}}} . \tag{26}$$

We estimate Eq. (26) by the method of least squares to obtain

$$\frac{E}{\sqrt{D}} = \underset{(44.9)}{.047} \left[\frac{(Y + S + F)}{\sqrt{D}} \right] + \underset{(23.6)}{205} \left(\frac{P_e A}{\sqrt{D}} \right) - \underset{(-22.5)}{63.6} \left(\frac{P_x N}{\sqrt{D}} \right)$$

$$R^2 = .98 \qquad s^2 = 1{,}821{,}000 . \tag{27}$$

The numbers in parentheses beneath the estimated coefficients are t-scores. All variables except P_e and P_x are measured in natural units. These price indices were given the value 1 for the academic year 1953-54.

The implied expenditure function at the school district level is

$$E = .047(Y + S + F) + 205P_e A - 63.6P_x N , \tag{28}$$

and the implied demand function at the school district level is

$$\frac{Q_e}{A} = 205 + .047\left[\frac{(Y + S + F)}{P_e A}\right] - 63.6\left(\frac{P_x N}{P_e A}\right), \qquad (29)$$

where S and F are state and federal aid equivalent to unrestricted cash grants.

From the least-squares estimates of the α's reported in Eq. (27) and the relationship between the parameters of the indifference map, Eq. (1), and the expenditure function, Eq. (4), we tentatively concluded that the indifference map of school districts is

$$U = \left[\left(\frac{Q_e}{A}\right) - 215\right]^{.047}\left[\left(\frac{Q_x}{N}\right) - 1362\right]^{.953}. \qquad (30)$$

Comparing Eqs. (28), (29), and (30) with Eqs. (11), (12), and (13), we see that the estimated coefficients are little affected by our correction for heteroskedasticity.

We now use the residuals from Eq. (27) to test some of the assumptions of the underlying stochastic model.

We have assumed that the error term in Eq. (26) has a mean equal to zero for all values of the independent variables. If this is not true, then the systematic part of the stochastic relationship is not a linear function through the origin. Since the errors are assumed to be identically distributed for all values of the independent variables, we will investigate the possibility of significant nonlinearities by dividing the range of values of the independent variables into subsets and by testing the hypothesis that the residuals in each subset are from a population with mean zero. This test is only approximate because we use the residuals that we observe rather than the errors that we cannot observe.

Table 3 contains the mean values of the residuals and the values of the test statistic.* There are 42 observations in each cell, so

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*For a description of this test, see A. M. Mood, *Introduction to the Theory of Statistics*, McGraw-Hill Book Company, Inc., 1950, pp. 259-261.

we reject the null hypothesis of mean equal to zero at the 1 percent level of significance if the value of the test statistic is outside the interval (-2.7, 2.7). Our assumption is rejected in three of the eight cases.

We have also assumed that the variance of the error term in Eq. (26) is the same for all values of the independent variables. We will investigate the possibility of heteroskedasticity by testing the hypothesis that the residuals in each subset of values of the independent variables specified in Table 3 are from populations with the same variance. Again, this test is approximate because we use residuals rather than errors.

As indicated in Table 4, there are large differences in the sample variances of the residuals at different values of the independent

Table 3

MEAN VALUES OF RESIDUALS AND VALUES OF TEST STATISTICS, ORIGINAL MODEL

$\frac{(Y + S + F)}{\sqrt{D}}$	Low $\frac{P_x N}{\sqrt{D}}$		High $\frac{P_x N}{\sqrt{D}}$	
	Low $\frac{P_e A}{\sqrt{D}}$	High $\frac{P_e A}{\sqrt{D}}$	Low $\frac{P_e A}{\sqrt{D}}$	High $\frac{P_e A}{\sqrt{D}}$
Low	238,000 (8.76)[a]	336,000 (3.70)[a]	103,000 (.68)	100,000 (.30)
High	38,000 (.72)	252,000 (1.48)	-1,056,000 (-4.16)[a]	-62,000 (-.10)

NOTES: The elements of these cells were determined as follows: The 168 residuals with the smallest values of $P_x N/\sqrt{D}$ were grouped into one subset and the other 168 residuals into another. For each of these subsets, the 84 residuals with the smallest values of $P_e A/\sqrt{D}$ were grouped into one subset, and the other 84 residuals into another subset. For each of these four subsets, the 42 residuals with the smallest values of $(Y + S + F)/\sqrt{D}$ were grouped into one subset; the 42 residuals with the largest values of $(Y + S + F)/\sqrt{D}$ were grouped into another subset.

[a]The test statistic is significantly different from zero at the .01 level.

variables. Under the null hypothesis of equal variances, the statistic

$$\frac{-2[\ln(c)]}{1 + \left[\frac{1}{3}(k-1)\right]\left[\Sigma\left(\frac{1}{n_i}\right) - \left(\frac{1}{\Sigma n_i}\right)\right]},$$

where $$c = \frac{\prod \left[\frac{S_i}{(n_i - 1)}\right]^{(n_i-1)/2}}{\left[\frac{\Sigma S_i}{\Sigma(n_i - 1)}\right]^{\Sigma(n_i-1)/2}},$$

$$S_i = \sum_{j=1}^{n_i} (r_{ij} - \bar{r}_i)^2,$$

r_{ij} = the j^{th} residual in the i^{th} cell,

$\bar{r}_i$ = the mean of the residuals in the i^{th} cell,

n_i = the number of residuals in the i^{th} cell, and

k = the number of cells.

has approximately the chi-square distribution with k - 1 degrees of freedom.* In this case, we will reject the null hypothesis at the 1-percent level of significance if the value of the test statistic exceeds 18.5. For this sample, the value of the test statistic is 559.3, thus leaving little doubt that the variance of the error term is not the same at all values of the independent variables.

Studies of expenditure on other goods generally show that the variance in expenditure increases with income. Hence, we expect the variance of the error term in a stochastic model explaining variation in expenditure in terms of variation in income to increase with income. After completing our analysis of the residuals from this regression, we analyze the residuals from the regression, Eq. (10), which takes this phenomenon into account.

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*For a description of this test, see Mood, op. cit., pp. 269-270.

Table 4

VARIANCES OF RESIDUALS, ORIGINAL MODEL
(in millions)

$\frac{(Y + S + F)}{\sqrt{D}}$	Low $\frac{P_x N}{\sqrt{D}}$		High $\frac{P_x N}{\sqrt{D}}$	
	Low $\frac{P_e A}{\sqrt{D}}$	High $\frac{P_e A}{\sqrt{D}}$	Low $\frac{P_e A}{\sqrt{D}}$	High $\frac{P_e A}{\sqrt{D}}$
Low	31,000	347,000	953,000	4,581,000
High	116,000	1,221,000	2,709,000	15,570,000

Finally, we consider an implication of our assumptions for the pattern of residuals of special interest to this study. Under our assumptions, the probability of positive error is one-half. Since the errors are assumed to be independent, the probability of positive errors for all seven years in any one state is 1 in 128. The probability of negative errors in all seven years is the same. Therefore, the probability of either all positive or all negative errors is 2 in 128. We expect less than 2 percent of the states to have either all positive or all negative residuals. For our sample, this phenomenon occurs in 40 percent of the states. Thus, it seems likely that Eq. (27) would consistently produce predictions that were too large in a significant number of states and too small in many others.

The Revised Model

We now examine the residuals from the regression, Eq. (10), to determine whether or not the assumptions of the revised stochastic model, Eqs. (5) and (6), are more nearly satisfied than those of the original model.

Table 5 contains the mean values of the residuals and the values of the test statistic for testing the hypothesis that each population mean is zero. We reject this hypothesis at the 1-percent level of significance if the value of the test statistic is outside the

interval (-2.7, 2.7). Therefore, our assumption is rejected in two of the eight cases. A comparison of Table 5 with Table 3 suggests that the assumption of linearity of the systematic part of the stochastic relationship is violated to about the same extent in the revised and original model.

Table 6 reports the variance of the residuals for the ranges of values of the independent variables specified in Table 5. As indicated in Table 6, there are still large differences in the sample variances of the residuals at different values of the independent variables. Formally, we reject the null hypothesis of equal variances at the 1-percent level of significance if the value of the test statistic exceeds 18.5. In this case, the value of the test statistic is 93.8, thus leaving little doubt that the variance of the error term is not the same for all values of the independent variables. However, it is also

Table 5

MEAN VALUES OF RESIDUALS AND VALUES OF TEST STATISTICS, REVISED MODEL

	Low $\frac{P_x N}{\sqrt{Y + S + F}}$		High $\frac{P_x N}{\sqrt{Y + S + F}}$	
$\sqrt{Y + S + F}$	Low $\frac{P_e A}{\sqrt{Y + S + F}}$	High $\frac{P_e A}{\sqrt{Y + S + F}}$	Low $\frac{P_e A}{\sqrt{Y + S + F}}$	High $\frac{P_e A}{\sqrt{Y + S + F}}$
Low	124.0 (4.12)[a]	72.0 (2.12)	126.4 (3.08)[a]	-116.9 (-2.67)
High	35.3 (1.16)	51.7 (1.29)	-42.7 (-.74)	10.4 (.12)

NOTES: The elements of these cells were determined as follows: The 168 residuals with the smallest values of $P_x N/\sqrt{Y + S + F}$ were grouped into one subset and the other 168 residuals into another. For each of these subsets, the 84 residuals with the smallest values of $P_e A/\sqrt{Y + S + F}$ were grouped into one subset, and the other 84 residuals into another subset, For each of these four subsets, the 42 residuals with the smallest values of $\sqrt{Y + S + F}$ were grouped into one subset; the 42 residuals with the largest values of this variable were grouped into another subset.

[a]The test statistic is significantly different from zero at the .01 level.

Table 6

VARIANCES OF RESIDUALS, REVISED MODEL
(in millions)

	Low $\frac{P_e A}{\sqrt{Y + S + F}}$		High $\frac{P_e A}{\sqrt{Y + S + F}}$	
$\sqrt{Y + S + F}$	Low $\frac{P_e A}{\sqrt{Y + S + F}}$	High $\frac{P_e A}{\sqrt{Y + S + F}}$	Low $\frac{P_e A}{\sqrt{Y + S + F}}$	High $\frac{P_e A}{\sqrt{Y + S + F}}$
Low	38,000	43,000	71,000	81,000
High	39,000	68,000	142,000	337,000

clear that the revision of the original model moved us in the right direction by a significant amount. This suggests that a stronger correction (e.g., assuming that the standard deviation of the error term is proportional to income) would eliminate heteroskedasticity. However, because of the necessity of aggregating from the school district level to the state level in order to use the available data, it is not possible to make a stronger correction. Fortunately, the results of the first revision suggest that parameter estimates would not be changed much by applying a stronger correction. So the remaining heteroskedasticity does not prevent us from using the estimated indifference map, Eq. (13).

If the assumptions of our revised model are correct, then we expect less than 2 percent of the states to have either all positive or all negative residuals. For the revised version of the model, this phenomenon occurs in 43 percent of the states. On this score, our assumptions are about equally violated in the original and revised versions of the model. Thus, it appears that Eq. (10) will consistently produce predictions that are too large in a significant number of states and too small in many others.

This outcome may be due in part to violations of the assumption that all state aid is equivalent to an unrestricted cash grant. Aid formulas in some states may be more stimulative than unrestricted cash

grants, though we have already argued that aid to the vast majority of school districts *is* equivalent to an unrestricted cash grant. In any event, the good fit of the estimated equation suggests that the magnitude of the prediction errors will not be large. Furthermore, in predicting the difference in total expenditure under alternative grant schemes, the bias will tend to cancel out.

TESTS OF SOME IMPLICATIONS OF THE REVISED MODEL

To increase our knowledge of the extent to which our model simulates reality, we will conduct tests of its implications. Since these tests are conditional on the assumptions investigated in the preceding discussion ("Tests of Some Assumptions of the Stochastic Models"), we can only hope that the tests are robust with respect to violations of these assumptions.

Tests of Equality of Coefficients in Cross-Sectional Relationships

We assume that the model underlying the estimated Eq. (10) is equally correct for explaining differences in expenditures between different school districts and changes in expenditures within a district over different years. Therefore, if we estimate Eq. (9) separately for each year, we expect to find no significant differences in the vectors of estimated coefficients of determination and estimated variances of the error terms. The t-scores are in parentheses.

We can test the hypothesis that the vectors of all coefficients in the cross-sectional relationships are the same by means of an analysis of covariance.* If this hypothesis is correct, there will be only one chance in a hundred that the test statistic

$$\frac{\left[\frac{(P - Q)}{(T - 1)K}\right]}{\left[\frac{Q}{(N - TK)}\right]}$$

*Tests of this kind were made known to economists by G. C. Chow ("Tests of Equality between Sets of Coefficients in Two Linear Regressions," *Econometrica*, Vol. 28, July 1960, p. 591-605). J. Johnson (*Econometric Methods*, McGraw-Hill Book Company, Inc., New York, 1963, p. 136-138) presents a lucid derivation of the test and a simple method of computing the value of the test statistic in the two-regression case. Many other economists and statisticians have derived similar tests.

would be greater than 1.94,

where P = the sum of squared residuals from the regression using data from all states and all years,

Q = the sum of the sums of squared residuals from the cross-sectional regressions,

T = the number of cross-sectional regressions,

K = the number of parameters to be estimated in the pooled regression, and

N = the total number of observations used for the pooled regression.

Table 7

CROSS-SECTIONAL REGRESSIONS

Year	Parameter Estimates			Measures of Fit	
	a_0	a_1	a_2	R^2	s^2
1953-1954	.029 (9.01)	175.23 (4.43)	-32.08 (-2.97)	.94	46,670
1955-1956	.032 (8.78)	165.95 (3.96)	-34.83 (-2.67)	.94	61,330
1957-1958	.038 (11.5)	225.06 (5.54)	-54.71 (-4.29)	.95	75,170
1959-1960	.044 (12.2)	223.01 (5.60)	-62.87 (-4.59)	.96	79,990
1961-1962	.046 (13.4)	194.87 (5.31)	-58.38 (-4.34)	.97	88,420
1963-1964	.044 (10.8)	116.52 (2.63)	-35.15 (-2.05)	.96	143,100
1965-1966	.044 (11.0)	123.31 (2.73)	-33.66 (-1.90)	.96	166,600

NOTES: Regressions were estimated from the following equation:

$$\frac{E}{\sqrt{Y + S + F}} = a_0\sqrt{Y + S + F} + a_1\left(\frac{P_e A}{\sqrt{Y + S + F}}\right) + a_2\left(\frac{P_x N}{\sqrt{Y + S + F}}\right)$$

Numbers enclosed in parentheses are t-scores.

The value of the test statistic in this case is 3.59. Therefore, we reject the hypothesis that the observations were generated by the stochastic model, Eqs. (5) and (6), in each year.

Several other features of Table 5 should be noted. First, it is clear that the variance of the error term is larger in later years.* This is probably another manifestation of the greater variability of educational expenditure at higher incomes. As mentioned in the preceding section, our data permit only a partial correction for this violation of one assumption of the classical linear regression model. Second, almost all coefficients in the cross-sectional regressions of our models are smaller in absolute value than the corresponding coefficients in the pooled regression.**

Tests of Equality of Coefficients of Income, State Aid, and Federal Aid

Our theory implies that an increase of one dollar in the disposable income of residents of the school district, in the amount of unrestricted state aid to the school district, or in the amount of unrestricted federal aid to the school district will all have the same effect on educational expenditure. This implication can be tested. The stochastic model underlying the estimated Eq. (10) implies that $b_0 = b_1 = b_2$ in the stochastic relationship

$$\frac{E_{st}}{\sqrt{(Y + S + F)_{st}}} = b_0\left[\frac{Y_{st}}{\sqrt{(Y + S + F)_{st}}}\right] + b_1\left[\frac{S_{st}}{\sqrt{(Y + S + F)_{st}}}\right] + b_2\left[\frac{F_{st}}{\sqrt{(Y + S + F)_{st}}}\right] + b_3\left[\frac{P_{e,t}A_{st}}{\sqrt{(Y + S + F)_{st}}}\right] + b_4\left[\frac{P_{x,t}N_{st}}{\sqrt{(Y + S + F)_{st}}}\right] + U_{st} . \quad (31)$$

*It should be mentioned that one assumption underlying the test of the preceding paragraph is that the variance of the error term is the same in each regression.

**W. W. McMahon, in his "An Economic Analysis of Major Determinants of Expenditures on Public Education," *Review of Economics and Statistics*, Vol. LII, August 1970, pp. 242-252, has sought to explain this phenomenon with a model different from the one presented in this report.

Table 8 reports the results of the pooled and cross-sectional regressions of this form. The t-scores are given in parentheses.

If the coefficients of income, state aid, and federal aid were the same, then the statistic

$$\frac{\frac{(P - Q)}{J}}{\frac{Q}{(T - K)}}$$

would have an F distribution with J and T - K degress of freedom,

where P = the sum of squared residuals from the regression in which the coefficients of income, state aid, and federal aid are constrained to be equal,

Q = the sum of squared residuals from the regression in which they are not constrained to be equal,

J = the number of linearly independent restrictions,

T = the number of observations, and

K = the number of parameters to be estimated in the unconstrained regression.

In this case, there are two linearly independent restrictions, namely

$$b_0 - b_1 = 0 \quad \text{and} \quad b_0 - b_2 = 0 ,$$

and five parameters to be estimated. There are 336 observations in the pooled regression and 48 observations in each cross-sectional regression. Hence, if the null hypothesis is correct, there is only one chance in a hundred that the test statistic will exceed 4.61 for the pooled regression or 5.22 for a cross-sectional regression. The values of the test statistic for this sample and its subsamples appear in the last column of Table 8.

The hypothesis of equality of the coefficients of income, state aid, and federal aid is emphatically rejected in the pooled regression. It is entirely clear that an increase of one dollar in the disposable income of the residents of a school district has less effect on educational expenditure in a school district than an increase of one dollar in the amount of state aid. A similar statement regarding federal

aid can be made with less certainty. We reject the null hypothesis in only two of the seven cross sections. However, the estimated coefficients of state and federal aid are greater than the estimated coefficient of income in each case.

We have already argued that state aid is equivalent to a price subsidy, rather than an unrestricted cash grant, for a small proportion of the school districts in our sample. Outside aid is more stimulative than unrestricted cash grants in these districts. For this reason, we expect the coefficient of state aid to be slightly larger than the coefficient of income in Eq. (31). Nevertheless, it does not seem

Table 8

POOLED AND CROSS-SECTIONAL REGRESSIONS WITH SEPARATE REGRESSORS FOR INCOME, STATE AID, AND FEDERAL AID

Year	Parameter Estimates					Measures of Fit		Test Statistic
	b_0	b_1	b_2	b_3	B_4	R^2	s^2	F ratio
Pooled regression	.041 (40.9)	.272 (8.81)	.171 (1.02)	180.87 (16.0)	-56.54 (-20.7)	.97	92,500	28.29
1953-1954	.026 (7.55)	.228 (2.39)	.176 (0.28)	111.79 (2.30)	-20.74 (-1.79)	.95	43,520	2.63
1955-1956	.029 (7.29)	.206 (2.26)	.788 (1.13)	89.22 (1.62)	-20.64 (-1.44)	.95	57,890	2.34
1957-1958	.034 (10.9)	.285 (3.81)	1.174 (1.62)	115.75 (2.42)	-35.30 (-2.83)	.97	58,530	7.39
1959-1960	.040 (11.2)	.230 (3.08)	.809 (1.25)	148.93 (3.12)	-48.97 (-3.52)	.97	71,150	3.79
1961-1962	.044 (13.2)	.230 (3.18)	.606 (0.77)	142.90 (3.26)	-50.73 (-3.84)	.97	79,460	3.54
1963-1964	.043 (11.0)	.279 (3.64)	.654 (0.80)	63.69 (1.33)	-29.73 (-1.87)	.97	121,300	5.04
1965-1966	.042 (8.20)	.332 (4.60)	.137 (0.25)	87.50 (2.13)	-34.39 (-1.88)	.97	126,200	8.22

NOTE: Numbers in parentheses are t-scores.

reasonable to attribute differences of the magnitude reported in Table 8 to this cause.

In this section we have tested two implications of the assumptions of our stochastic model, and in both cases found these implications inconsistent with our data.

VI. FINAL EVALUATION OF THE MODEL AND SUGGESTIONS FOR FURTHER RESEARCH

The purpose of this report is to provide a method for making an estimate of the effect of any form and amount of outside aid to a school district on its educational expenditure. None of the many empirical studies of the determinants of expenditures on education by local school boards are suitable for this purpose because they either ignore the effect of outside aid or implicitly assume that the form of aid makes no difference.* Therefore, a statistical comparison of our estimated expenditure equation with other estimated relationships is not relevant to the purpose of this study.** However, it is perhaps

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*This statement is true for at least the following studies: E. F. Renshaw, "A Note on the Expenditure Effect of State Aid to Education," *Journal of Political Economy*, Vol. 68, April 1960, pp. 170-174; W. Z. Hirsch, "Income Elasticity of Public Education," *International Economic Review*, Vol. 2, September 1961, pp. 330-339; R. Kosobud, "Forecasting Public Education Expenditures," *Papers and Proceedings of the Regional Science Association*, Vol. 11, 1963, pp. 253-284; G. W. Fisher, "Interstate Variation in State and Local Expenditure," *National Tax Journal*, Vol. 17, March 1964, pp. 57-74; S. Sacks and R. Harris, "The Determinants of State and Local Expenditures and Intergovernmental Flows of Funds," *National Tax Journal*, Vol. 17, March 1964, pp. 75-85; G. A. Bishop, "Stimulative versus Substitutive Effects of State School Aid in New England," *National Tax Journal*, Vol. 17, June 1964, pp. 133-143; W. W. McMahon, "An Economic Analysis of Major Determinants of Expenditures on Public Education," *Review of Economics and Statistics*, Vol. 52, August 1970, pp. 242-252; T. O'Brien, "Grants-in-Aid: Some Further Answers," *National Tax Journal*, Vol. 24, March 1971, pp. 65-78; E. M. Gramlich, "A Comment on O'Brien's 'Grants-in-Aid'," *National Tax Journal*, Vol. 25, March 1972, pp. 107-108; G. S. Tolley and E. Olson, "The Interdependency between Income and Education," *Journal of Political Economy*, Vol. 79, May-June 1971, pp. 460-480. There are theoretical models in the literature which predict that the form of outside aid does affect educational spending. For example, see J. A. Wilde, "The Expenditure Effects of Grant-in-Aid Programs," *National Tax Journal*, Vol. 21, September 1968, pp. 340-348; and D. F. Bradford and W. E. Oates, "Towards a Predictive Theory of Intergovernmental Grants," *American Economic Review*, Vol. 51, May 1971, pp. 440-448.

**It should be noted that the reported coefficients of determination should not be used to compare the goodness of fit of estimated relationships having different dependent variables (e.g., expenditure versus expenditure per capita) and using data aggregated to different levels (e.g., school district versus state). See H. Theil, *Principles of Econometrics*, John Wiley and Sons, Inc., 1971, pp. 181 and 542-545.

useful to comment on the extent to which our theory is confirmed by the data and to offer some suggestions for further research.

The theory developed in this report has led to a stochastic model which permits us to explain a very high proportion of the variation in current educational expenditure divided by the square root of the sum of personal disposable income and state and federal aid to school districts, using data aggregated to the state level. The theory also has certain implications concerning the signs and in one case the magnitude of the regression coefficients. These implications were stated as hypotheses, and the data led us to accept each of these hypotheses.

An analysis of the residuals from this regression revealed significant deviations from the assumed functional form of the systematic part of the stochastic relationship over only two of eight ranges of the regressors. It also indicated that our correction for heteroskedasticity was not strong enough. If data at the school district level had been used, we could have assumed that the standard deviation of the error term was proportional to income. This specification would probably eliminate heteroskedasticity and result in little change in our estimates of the parameters of the indifference map. Lastly, the analysis of residuals suggested that our estimated expenditure equation would consistently produce predictions that are too large in a significant number of states and too small in many others. This is not regarded as an important defect from the standpoint of predicting the *difference* in educational expenditure in a school district which would result from a difference in the form of outside aid. Nevertheless, if data at the school district level had been used, we could have written the parameters of the indifference map as functions of school district characteristics (e.g., mean years of education of adults) that might be expected to result in different preferences in different school districts, and we would have written the price indices as functions of our national indices and characteristics of school districts (e.g., population density) that might be expected to result in geographical differences in prices.

In the preceding section we conducted two powerful tests of our model. The model implies that the vector of coefficients of the expenditure equation is the same in all years. The data are inconsistent with this hypothesis. This suggests that important determinants of educational expenditure have been omitted from the model and that these determinants have not remained constant over time. It is possible that a modification of the model along the lines suggested in the preceding paragraph would eliminate this source of discrepancy between the data and the implications of the model. Our model also implies that the coefficients of income, state aid, and federal aid in the expenditure equation are equal. The data are clearly inconsistent with this hypothesis. In justifying our estimating equation, we argued that almost all past state and federal aid to local school districts is equivalent to unrestricted cash grants, but we recognized that some outside aid has been more stimulative than these grants. Hence, we expected that the coefficients of state and federal aid would be somewhat larger than the coefficient of income. However, the magnitudes of these differences seem inconsistent with the assumptions of our model, including the assumption concerning the effective form of outside aid during the time period under consideration. In future studies, more attention should be devoted to determining the effect of existing state and federal grants on the budget constraints of local school districts and the consequences of these budget constraints for estimating expenditure relationships.

It is not surprising that the theory developed in this report fails powerful tests because it is essentially metaphorical. The metaphor partially captures one extremely important aspect of reality, namely, that the quantities of educational service consumed by an individual and (in many cases) by others represent only several of many goods that the individual desires. Thus, there is a limit to how many of the other goods an individual is willing to sacrifice to obtain an additional amount of education for himself or for others. Expenditure on education is not four times its present level because *virtually no one* values the increased education that he would receive more than the other goods that he would have to forego. Nevertheless, the theory

does not explain how individual preferences and constraints combine with existing institutional arrangements to produce collective decisions on educational expenditures.

There are theories in the literature less metaphorical in their assumptions about individual behavior and the interactions of individuals in the collective decisionmaking process.* In the long run, these models will surely lead to better predictions of the effects of different forms of outside aid on local educational expenditure.

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*In addition to the paper by Bradford and Oates, ibid., see J. L. Barr and O. A. Davis, "An Elementary Political and Economic Theory of the Expenditures of Local Governments," *Southern Economic Journal*, Vol. 33, October 1966, pp. 149-165, and S. M. Barro, *Theoretical Models of School District Expenditure Determination and the Impact of Grants-in-Aid*, The Rand Corporation, R-867-FF, February 1972.